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DESIGN OF SMALL BULB TURBINES WITH UNEQUAL SPECIFIC WORK DISTRIBUTION OF THE RUNNER'S ELEMENTARY STAGES

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Abstract. *Regarding the present state of knowledge in the field of the turbomachinery design, the method for designing small bulb turbines with unequal specific work distribution of the turbine runner's elementary stages near the hub is presented in the paper. The distribution function of specific work of all the elementary stages is obtained, according to which the averaged axisymmetric flow surfaces of the turbine runner have a negligibly small deviation from the cylindrical flow surfaces. The specific work of the near-the-hub elementary stages, in the given distribution function, can be reduced up to 60% of the required (design) specific work, still achieving nearly cylindrical flow surfaces.*

Key Words: *Bulb Turbine, Elementary Stages, Specific Work, Axisymmetric Flow Surfaces*

1. INTRODUCTION

Both the hydraulic turbines design and their operating performance analysis require the use of simpler methodologies in preliminary design phases, especially when the geometry of the turbine runner is not completely defined. In the field of the turbomachinery design, such attempts are continuously made thus providing us with different methodologies and procedures for the design process optimization [10, 11]. With the computer technology development, a significant breakthrough of numerical methods and numerical simulation of the fluid flow has been made, encouraging the numerical techniques incorporation into the design procedure and performance analysis of the turbomachinery [12, 13].

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A number of assumptions and simplifications, empirical equations as well as the designer's experience always accompany the designing process [1, 2, 5, 7, 9]. The basic assumption in the hydraulic turbine designing procedure is axisymmetric flow surfaces. An elementary stage is a flow space between two elementary immediate axisymmetric flow surfaces. The intersection of stay vane (SV), or turbine runner (TR), and axisymmetric flow surfaces, defines the vane or blade profiles in the turbine elementary stages.

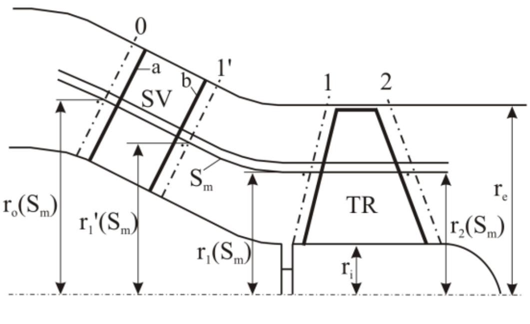
Stay vanes and blades are formed by the stay vane profiles and the runner blade profiles in the turbine's elementary stages. The vane and blade profiles in the turbine elementary stages can be defined using suitable hydraulic calculations for a two-dimensional fluid flow through the profile cascade of axisymmetric flow surfaces. In the cases when the flow surfaces are cylindrical, the theory of fluid flow through the straight plane profile cascade can be used in calculations [6].

In order to achieve cylindrical or almost cylindrical flow surfaces in the bulb turbine runners, these runners are usually designed to obtain an equal specific work of all the elementary stages. The turbine runners designed according to such a principle are made of blade profiles with a significantly larger inclination angle near the hub than near the shroud.

In order to minimize the blades' spatial curvature and to reduce the axial length of the runner, it is eligible to design the runner with smaller specific work of the elementary stage nearer the hub than the blade periphery [3, 4]. In this way, the spatial curvature of the stay vanes is also reduced. The possibility to design such stay vanes and turbine runners is analyzed in the paper. Only small bulb turbines, which can be used in small hydropower plants or as models of large water turbines (for the model testing purpose) are considered in this study.

2. BASIC FORMULAS

The scheme of a bulb turbine meridional cross-section, with traces of two elementary immediate axisymmetric flow surfaces, where one (below) is marked as S_m, is presented in Fig. 1. A flow space between two elementary immediate axisymmetric flow surfaces represents an elementary stage of the runner. To determine the shape of stay vanes and runner blades, it is sufficient (for small turbines) to define profiles in 7 to 12 elementary stages, approximately evenly distributed along the blade height.



TR - turbine runner
SV - stay vane
S_m - flow surface of elementary flow through the turbine
0, 1'- inlet and outlet control cross-section of SV
1, 2- inlet and outlet control cross-section of SV
$r(S_m)$ - radius of various control cross-sections

Fig. 1 Scheme of a bulb turbine meridional cross-section

The runner is designed as a free-vortex flow (to obtain zero circumferential component of absolute flow velocity at outlet (c_{u2}=0)); therefore, based on the Euler's equation, the specific work of the turbine runner elementary stage is:

$$y_k(S_m) = \omega \cdot r_1(S_m) \cdot c_{u1}(S_m)\,, \quad \text{za } c_{u2}(S_m) = 0\,, \tag{1}$$

where: c_u, c_r, c_z – circumferential, radial and axial component of absolute flow velocity [m/s] (absolute flow velocity $\vec{c} = c_u \vec{u}^{\,o} + c_r \vec{r}^{\,o} + c_z \vec{z}^{\,o}$), r_1 – radius in cross-section 1 [m], ω– angular velocity of turbine runner [s^{-1}].

According to Eq. (1), the required circumferential component of absolute flow velocity, in front of the inlet of the turbine elementary stage, can be defined by formula:

$$c_{u1}(S_m) = \frac{y_k(S_m)}{\omega \cdot r_1(S_m)}\,. \tag{2}$$

Stay vanes produce circumferential components of absolute velocity in front of the runner inlet. Neglecting the influence of viscous friction, the flow in area between SV and TR (vaneless area) is free-vortex flow:

$$r(S_m) \cdot c_u(S_m) = const. = r_{1'}(S_m) \cdot c_{u1'}(S_m) = r_1(S_m) \cdot c_{u1}(S_m)\,,$$

therefore, the circumferential components of absolute velocity of the stay vane elementary stages can be calculated as:

$$c_{u1'}(S_m) = \frac{r_1(S_m)}{r_{1'}(S_m)} c_{u1}(S_m) = \frac{y_k(S_m)}{\omega \cdot r_{1'}(S_m)}\,. \tag{3}$$

Streamline inclination angle (with respect to circumferential coordinate $r\varphi \cdot \vec{u}^{\,o}$, where $\vec{u}^{\,o}$ is the unit vector in the direction of circumferential velocity $\vec{u}$, $\vec{u} = u \cdot \vec{u}^{\,o}$, $u = r \cdot \omega$) in the outlet of stay vane area is calculated by the next formula:

$$\alpha_{1'}(S_m) = arctg\, \frac{c_{m1'}(S_m)}{c_{u1'}(S_m)}\,, \quad \text{where } c_m = \sqrt{c_r^2 + c_z^2}\,. \tag{4}$$

To determine the inclination angles of vane profile chamber line, in the outlet of the stay vane elementary stages, $\alpha_{1.b}(S_m) = \alpha_{1'}(S_m) - \Delta\alpha_{b.1'}(S_m)$, it should be assumed that the value of flow inclination angle in outlet of the stay vane is ($\Delta\alpha_{b.1'}(S_m) = (2 \div 5)^o$).

In front of the inlet of the stay vane, there is a zero circumferential component of the absolute velocity (c_{u0} = 0), thus, the inclination angle of the camber line in the inlet of the turbine stay vane is $\alpha_{1.a}(S_m) = 90^o$.

Design operating parameters of the turbine are: ω, $Q = Q^+$, $Y_T = Y_T^+$, where Q^+ and Y_T^+ are the best efficiency (optimal) operating parameters (when the turbine operates with maximum efficiency, $\eta = \max\eta = \eta^+$), Q [m^3/s] is turbine volume flow rate and Y_T [J/kg] is turbine specific work ($Y_T = gH_T$ (g = 9.81 m/s^2), H_T – net turbine head [m]).

Adopting the maximum value of hydraulic efficiency ($\eta_h = \eta_h^+$), design specific work of the turbine runner is $Y_k = Y_k^+ = \eta_h^+\ Y_T^+$. When designing a turbine using the model of equal specific work of turbine elementary stages ($y_k(S_m)$ = const. = Y_k^+), and when flow surfaces in the runner are cylindrical ($r_1(S_m) = r_2(S_m)$, $c_{z1} = c_{z2}$ = const.), the theory of flow

in the straight plane profile cascades is used. Inclination angles of profile chord in elementary stages of the turbine runner ($\beta_t = \beta_t\,(S_m)$) are defined by the lift force method, where, using the above mentioned theory, the well known formula is used [1, 2]:

$$\xi_y \frac{l}{t} = 2\frac{\Delta w_u}{w_\infty}\,, \tag{5}$$

where: ξ_y – profile lift coefficient, l – length of the chord line, t – cascade spacing, $\Delta w_u = w_{u1} - w_{u2}$ ($\Delta w_u = \Delta c_u$, for $c_{u2} = 0$), $\vec{w} = \vec{c} - \vec{u}$ is relative velocity of water flow, $\vec{w} = w_u \vec{u}^o + w_r \vec{r}^o + w_z \vec{z}^o$ ($w_u = c_u - u$, $w_r = c_r$, $w_z = c_z$) and w_∞ – value of averaged velocity in infinity ($\vec{w}_\infty = 0.5(\vec{w}_1 + \vec{w}_2)$).

For elementary stages of the runner there are: $\xi_y = \xi_y\,(S_m)$, $t = t\,(S_m)$, $l = l\,(S_m)$, $\Delta w_u = \Delta w_u\,(S_m)$ and $w_\infty = w_\infty\,(S_m)$. Since coefficient ξ_y depends on inclination angle of the profile chord line, based on formula (5), for calculated value ξ_y, the inclination angle of the profile chord line can be determined respectively.

In the case where flow surfaces in the runner are not cylindrical (when flow components are not uniform along the radius), volume flow rate $Q\,(S_m)$ passing through the flow space between the hub and flow surface S_m, is calculated as follows:

$$Q(S_m) = 2\pi \int_{r_i}^{r_1(S_m)} c_{z1}(r) r dr = 2\pi \int_{r_i}^{r_2(S_m)} c_{z2}(r) r dr\,, \tag{6}$$

where, $c_{z1}(r)$ and $c_{z2}(r)$ are distribution functions of axial components of absolute flow velocities in the control cross-sections in front of and behind the runner. Functions $c_{z1}(r)$ and $c_{z2}(r)$ should satisfy the following formula of volume flow through the runner:

$$Q = 2\pi \int_{r_i}^{r_e} c_{z1}(r) r dr = 2\pi \int_{r_i}^{r_e} c_{z2}(r) r dr\,. \tag{7}$$

For given function $y_k\,(r)$, the specific work of turbine runner can be calculated using the formula:

$$Y_k = \frac{1}{Q}\int_{A_2} y_k(r)\cdot dQ = \frac{1}{Q}\left(2\pi \int_{r_i}^{r_e} y_k(r) c_{z2}(r) r dr\right). \tag{8}$$

3. Conditions for Obtaining Cylindrical Flow Surfaces in the Control Cross-Sections in Front of and Behind the Turbine Runner

The necessary condition is that the control cross-sections are placed in the space which is physically bounded by cylindrical surface of the hub (turbine hub radius r_i = const.) and the shroud (turbine shroud radius r_e = const.), as shown in Fig. 1.

The equation of steady fluid flow of the inviscid fluid, disregarding gravity acceleration, can be written in the form:

$$\left[\vec{c}, rot\vec{c}\right] = -\frac{1}{\rho} gradp_t \,, \tag{9}$$

where p is static pressure [Pa] and p_t the total pressure ($p_t = p + \rho c^2 / 2$) [Pa].

For the fluid flow on cylindrical surfaces ($c_r = 0$, $\vec{c} = c_u \vec{u}^o + c_z \vec{z}^o$) in control cross-sections in front of and behind the turbine runner (where $\partial / \partial \varphi = 0$), according to the component of vector Eq. (9) in the direction of coordinate r, a differential equation is obtained:

$$c_z \frac{\partial c_z}{\partial r} + \frac{c_u}{r} \frac{\partial (rc_u)}{\partial r} = \frac{1}{\rho} \frac{\partial p_t}{\partial r} \,, \tag{10}$$

which defines the condition for cylindrical flow surfaces in control cross-sections 1-1 and 2-2.

In control cross-section 2-2 (outlet of the runner) $c_{u2} = 0$ and $p_t = p_{t,2} =$ const., therefore, according to equation (10) it is $c_z = c_{z2} =$ const.

Defining the hydraulic efficiency of the runner elementary stage as:

$$\eta_{h.k}(S_m) = \frac{\rho \cdot y_k(S_m)}{\Delta p_{t_{1-2}}(S_m)} = \frac{\rho \cdot y_k(S_m)}{p_{t_1}(S_m) - p_{t_2}(S_m)} \,. \tag{11}$$

and, assuming that hydraulic efficiency is the same for all elementary stages ($\eta_{h.k}(S_m) = const. = \eta_{h.k}$), the total pressure in control cross-section 1-1 (inlet of the runner) can be expressed as:

$$p_{t_1}(r) = p_{t_2} + \frac{\rho y_k(r)_1}{\eta_{h.k}} \,. \tag{12}$$

According to Eq. (1):

$$rc_{u_1} = \frac{1}{\omega} y_k(r)_1 \,, \quad \text{i.e.} \quad c_{u_1} = \frac{1}{r\omega} y_k(r)_1 \tag{13}$$

where $y_k(r)_1 = y_k(S_m)$, due to $r_1 = r_1(S_m)$.

Considering Eqs. (12) and (13), Eq. (10) for obtaining cylindrical surfaces in the control cross-sections 1-1 (for $p_{t1}(r) = p_{t1}$, $c_z = c_{z1}$, $c_u = c_{u2}$) yields:

$$c_{z1} \frac{\partial c_{z1}}{\partial r} = \left(\frac{1}{\eta_{h.k}} - \frac{1}{r^2 \omega^2} y_k(r)_1 \right) \frac{\partial y_k(r)_1}{\partial r} \,. \tag{14}$$

where: $\dfrac{1}{\eta_{h.k}} - \dfrac{y_k(r)}{r^2 \omega^2} > 0$.

For equal specific work of the runner elementary stages ($y_k(r) = const = Y_k^+$), due to equation (14) it is obtained $c_{z1} =$ const., and due to Eqs. (6) and (7) $c_{z1} = c_{z2}$ and $r_1(S_m) = r_2(S_m)$, thus the assumption of the cylindrical flow surfaces in the runner is reasonable [8].

If $\partial y_k / \partial r > 0$, according to equation (14), $\partial c_{z1} / \partial r > 0$, thus, due to equation (5), for $c_{z2} = \text{const.} = \overline{c}_z$, can be resolved that $r_1(S_m) > r_2(S_m)$ and flow surfaces are approximately conical in the runner.

Further on, the relationship between the hydraulic efficiency of turbine runner ($\eta_{h.k}(S_m)$) and turbine hydraulic efficiency ($\eta_h(S_m)$) is obtained.

Denoting $p_{t.I}$ and $p_{t.II}$ as total pressures on inlet (I) and outlet (II) of turbine, the hydraulic efficiency of the turbine elementary stage is defined by relation:

$$\eta_h(S_m) = \frac{y_k(S_m)}{y_T(S_m)} = \frac{\rho y_k(S_m)}{p_{t.I}(S_m) - p_{t.II}(S_m)}. \tag{15}$$

Since $p_{t.I}(S_m) - p_{t.II}(S_m) = \Delta p_{t_{I-1}} + \Delta p_{t_{1-2}} + \Delta p_{t_{2-II}} = \rho y_{g_{I-1}} + \rho y_{g_{1-2}} + \rho y_{g_{2-II}}$, where $y_{g_{I-1}}$ and $y_{g_{2-II}}$ are specific losses of flow energy in the flow passages from I to 1 and the flow passages from 2 to II, it is easy to predict that there is a relationship between $\eta_h(S_m)$ and $\eta_{h.k}(S_m)$:

$$\eta_h = \frac{1}{\eta_{h.k}^{-1} + (\varepsilon_{I-1} + \varepsilon_{2-II})}, \quad \text{or} \quad \eta_{h.k} = \frac{1}{\eta_h^{-1} - (\varepsilon_{I-1} + \varepsilon_{2-II})}, \tag{16}$$

where, $\eta_h = \eta_h(S_m)$, $\eta_{h.k} = \eta_{h.k}(S_m)$, $\varepsilon_{I-1} = \varepsilon_{I-1}(S_m)$ and $\varepsilon_{2-II} = \varepsilon_{2-II}(S_m)$ are dimensionless losses of flow energy in the passages from *I* to 1 and the flow passages from 2 to *II* (turbine diffuser),

$$\varepsilon_{I-1} = \varepsilon_{I-1}(S_m) = \frac{y_{g.I-1}(S_m)}{y_k(S_m)} \quad \text{and} \quad \varepsilon_{2-II} = \varepsilon_{2-II}(S_m) = \frac{y_{g.2-II}(S_m)}{y_k(S_m)}. \tag{16'}$$

Since $\Delta p_{t_{1-2}}(S_m) = \rho y_k(S_m) + \rho y_{g.1-2}(S_m) = \rho(1+\varepsilon_{1-2}(S_m)) \cdot y_k(S_m)$, where $\varepsilon_{1-2}(S_m) = y_{g.1-2}(S_m)/y_k(S_m)$, based on Eq. (11), it can be written:

$$\eta_{h.k}(S_m) = \frac{1}{1+\varepsilon_{1-2}(S_m)}. \tag{17}$$

According to Eq. (16), it can be written:

$$\eta_h = \frac{1}{1+(\varepsilon_{I-1} + \varepsilon_{1-2} + \varepsilon_{2-II})}, \quad \text{i.e.} \quad \varepsilon_{I-1} + \varepsilon_{1-2} + \varepsilon_{2-II} = \frac{1}{\eta_h} - 1. \tag{17'}$$

Dimensionless loss ε_{I-1} contains flow energy losses of the stay vane. The designing assumption is that terms ε_{I-1} and ε_{1-2} are larger than dimensionless losses in the diffuser, ε_{2-II}.

4. SUGGESTION FOR FUNCTION $y_k(r)$

In order to reduce specific work of elementary stages near the hub, and to achieve flow surfaces inside the runner that do not deviate much from the cylindrical surfaces, the distribution function of elementary stages specific work is suggested as follows:

$$\left.\begin{aligned} & y_k(r) = A + 2r_o B \cdot r - B \cdot r^2, \quad \text{for} \quad r_i \le r \le r_0 \\ & \text{and} \quad y_k(r) = y_{k.0} = \text{const.}, \qquad \text{for} \quad r_0 \le r \le r_e \end{aligned}\right\} \tag{18}$$

where is $\partial y_k(r)/\partial r = 0$ for $r = r_0$.

Function $y_k(r)$ is defined according to radius r in the control cross-section 1-1 (in front of the runner).

Function graph $y_k(r)$, which is defined by equation (18) is presented in Fig. 2. Function $y_k(r)$ is defined according to accepted parameters r_o and $y_{k.i} = y_k(r_i)$. To obtain flow surfaces in the turbine runner with only a small deviation of the cylindrical surfaces the following is recommended:

$$r_0 \le \frac{1}{2}(r_i + r_e) \quad \text{and} \quad y_{k.i} = (0.6 \div 0.7) Y_k^+ ,$$

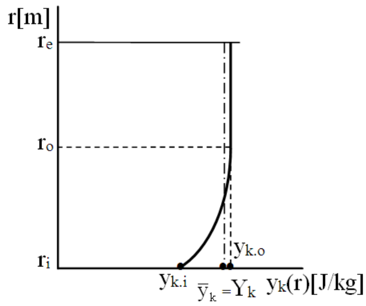


Fig. 2 Function $y_k(r)$

where Y_k^+ is design specific work of the runner.

The value of specific work $y_{k.o}$ is an unknown value and it can be defined using a requirement that specific work of runner (Y_k) obtained by Eq. (8), is equal to the design specific work of runner ($Y_k = Y_k^+$).

Value $y_{k.0}$ is determined, as will be shown later, using an iterative procedure. The coefficients A and B in the first Eq. (18) are calculated using formulae:

$$B = \frac{y_{k.0} - y_{k.i}}{(r_0 - r_i)^2} \quad \text{and} \quad A = y_{k.0} - r_0^2 B , \tag{19}$$

obtained from the requirement $y_k(r_i) = y_{k.i}$, for $r = r_i$ and $y_k(r_0) = y_{k.0}$, for r=r$_0$.

Assuming that the flow surfaces in the control cross-section in front of the runner (1-1) are cylindrical, based on Eqs. (14) and (18), it is obtained:

$$\left.\begin{aligned} &\frac{\partial c_{z1}^2}{\partial r} = \alpha - \beta \cdot r + \frac{\gamma}{r} - \frac{\delta}{r^2}, \quad \text{for} \quad r_i \le r \le r_0 \\ &\text{and} \quad c_{z1} = const. = c_{z1}(r_0), \quad \text{for} \quad r_0 \le r \le r_e \end{aligned}\right\} \tag{20}$$

where the coefficients are:

$$\alpha = 4 r_0 B\left(\frac{1}{\eta_h} + \frac{3B}{\omega^2}\right), \; \beta = 4B\left(\frac{1}{\eta_h} + \frac{B}{\omega^2}\right), \; \gamma = 4B\frac{A - 2r_0^2 B}{\omega^2} \quad \text{and} \quad \delta = \frac{4 r_0 AB}{\omega^2} \tag{21}$$

Integral of the first Eq. (20), for $r_i \le r \le r_o$, becomes:

$$c_{z1}(r) = \sqrt{(c_{z1}(r_o))^2 - \alpha(r_o - r) + \frac{1}{2}\beta(r_o^2 - r^2) - \gamma \ln\frac{r_o}{r} + \delta\left(\frac{1}{r} - \frac{1}{r_o}\right)} . \tag{22}$$

In the previously shown function $c_{z1}(r)$, for $r_i \le r \le r_0$, there is an unknown velocity value $c_{z1}(r_0)$ (axial component of flow velocity $c_{z1} = c_{z1}(r_0)$, for $r_0 \le r \le r_e$). This velocity can be defined according to the requirement that the flow rate calculated by Eq. (7) is equal to the design turbine flow rate ($Q = Q^+$). Since coefficients A and B, and also coefficients α, β, γ and δ, depend on unknown variable $y_{k.0}$, it can be concluded that function $c_{z1}(r_0)$ is determined using an iterative procedure.

Since $c_{z1}(r) = c_{z1}(r_0)$, for $r_0 \le r \le r_e$, and for $r_i \le r \le r_0$ $c_{z1}(r)$ is defined by Eq. (22), Eqs. (7) and (8), used for determination of values $y_{k.0}$ and $c_{z1}(r_0)$, can be derived into forms:

$$Q = 2\pi\int_{r_i}^{r_0} c_{z1}(r)rdr + \pi c_{z1}(r_0)(r_e^2 - r_0^2) \tag{23}$$

$$Y_k = \frac{1}{Q}2\pi\int_{r_i}^{r_0} y_k(r)c_{z1}(r)rdr + \frac{1}{Q}(\pi y_{k.0}c_{z1}(r_0)(r_e^2 - r_0^2))\,. \tag{24}$$

5. DETERMINATION OF $y_{k.0}$ AND $c_{z1}(r_0)$

Values $y_{k.0}$ and $c_{z1}(r_0)$ are determined by using an iterative procedure. Perhaps, it is better to say that this is a double iterative procedure, since in each step of determining $y_{k.0}$, $c_{z1}(r_0)$ should be determined as well by another iterative procedure [3].

In order to reduce the number of iterative steps for determining $y_{k.0}$, this value is obtained in the first iterative step, using a formula:

$$y_{k.0}^{(1)} = \frac{Y_k^+ - k \cdot y_{k.i}}{1-k}\,, \tag{25}$$

where $k = k\,(r_0, r_i, r_e)$ is:

$$k = \frac{r_0^2(r_0^2 - r_i^2) + 0.5(r_0^4 - r_i^4) - 1.333r_0(r_0^3 - r_i^3)}{(r_e^2 - r_i^2)(r_0 - r_i)^2}\,. \tag{25'}$$

Formula (25) is derived using the assumption that the axial components of flow velocities in the control cross-section in front of the runner change negligibly (for $c_{z1} = \text{const.} = \overline{c}_z$).

Using value $y_{k.0}^{(1)}$, which was determined by formula (25), the procedure of determining $y_{k.0}$ is completed in maximum of three iterative steps. The reason for this, as calculations showed, is that $c_{z1}(r)$ values change less than specific work $y_k(r)$, for $r_i \le r \le r_0$. In iterative steps of determining $y_{k.0}$ correction of this value is performed, requiring that calculated specific work (Y_k) using formula (24) slightly differs from the design specific work of the turbine runner Y_k^+ ($\left|Y_k - Y_k^+\right| / Y_k^+ \le 0.005$).

In each iterative procedure of determining $y_{k.0}$ the value $c_{z1}(r_0)$ is also obtained by an iterative procedure. In the first iterative step it can be taken that $c_{z1}(r_0) = 1.2\overline{c}_z$, where:

$$\overline{c}_z = Q/[\pi(r_e^2 - r_i^2)]\,, \quad \text{for} \quad Q = Q^+. \tag{26}$$

In iterative steps of determining $c_{z1}(r_0)$ a correction of this value is performed, and according to formula (23), calculated flow rate Q slightly differs from design flow rate Q^+ ($\left|Q - Q^+\right| / Q^+ \le 0.005$).

In the program for determining $y_{k.0}$ and $c_{z1}(r_0)$ [4], in the iterative procedure of solving the problem, $c_{z1}(r_0)$ changes with the iterative step $\pm 0.005 \cdot \overline{c}_z$, and $y_{k.0}$ changes with the

step $\pm 0.005 \cdot Y_k^+$. The integrals in formulae (23) and (24) are calculated using a trapezoid rule, whereas in the integration area from r_i to r_0 calculation points are distributed for the iterative step $\Delta r = 0.0025$m (2.5mm). Besides printing the calculated values $y_{k.0}$ and $c_{z1}(r_0)$, program [4] is also developed to print values $y_k(r)$, $c_{z1}(r)$ and $Q(r)$, for $r \in [r_i, r_e]$, for each iterative step $\Delta r = 0.0025$m, where r – radius in control cross-section in front of the turbine runner and $Q(r)$ is the volume flow rate under the cylindrical flow surface of radius r.

In the control cross-sections before (1-1) and behind (2-2) turbine runner, the flow surfaces are cylindrical, where $c_{z2} = \text{const.} = \bar{c}_z$. According to $Q(r)$ data, where $r = r_1(S_m)$, the functional relationship of cylinder radius in the same axisymmetric flow surfaces can be obtained. Regarding equation (6), for $Q(S_m) = Q(r)$, $r_2(S_m) = r_2(r)$ and $c_{z2} = \text{const.} = \bar{c}_z$ it follows:

$$r_2(r) = \sqrt{r_i^2 + \frac{Q(r)}{\pi \bar{c}_z}}\,, \tag{27}$$

where $r = r_1(S_m)$ and $r_2 = r_2(S_m)$.

For $r_2(r) < 1$, as obtained for $\partial y_k / \partial r > 0$, the flow surfaces in the turbine runner are approximately conical and $r_1(S_m) = r_2(S_m)$.

To determine the real flow surfaces' deviation from the cylindrical shape, the dimensionless function is $\bar{r}_{1/2}(r) = r / r_2(r)$. In cases where $\bar{r}_{1/2}(r) \le 1.03$, it can be said that the real flow surface in the turbine runner negligibly vary from the cylindrical surface.

The same calculation procedure has been used for determining values $y_{k.0}$ and $c_{z2}(r_0)$ and coefficients A, B, α, β, γ and δ like in the computer program given in Ref. [4]. Even though the program is originally created for axial flow fans, the same code can be used for bulb turbine as well, replacing value η_h with $1/\eta_h$ and value $c_{z2}(r_0)$ with $c_{z1}(r_0)$.

Diagrams of $y_k(r)$, $c_{z1}(r)$ and $\bar{r}_{1/2}(r)$ are given in Fig. 3, for $y_{k.i} = 0.7 Y_k^+$ and different values of r_0 (r_0 = 180, 210, 240, 280 and 310 mm) of one small bulb turbine, which is built in the small hydro power plant "Grcki mlin" near Prokuplje. The geometrical and design operating parameters of the turbine are: $r_i = 142$ mm, $r_e = 355$ mm, $Q^+ = 1.4$m^3/s, $Y_k^+ = 20.36$ J/kg and $\omega = 41.9$ r/s ($n = 400$ min^{-1}). For adopted $\eta = 0.75$ ($\eta_m = 0{,}94$, $\eta_Q = 0.96$ and $\varepsilon_{1\text{-}2} = \varepsilon_{I\text{-}1} + \varepsilon_{2\text{-}II}$) it follows that $\eta_{h.k} = 0.91$. As can be concluded from Fig. 3, for $y_{k.i} = 0.7 Y_k^+ = 14.2$ J/kg, $\bar{r}_{1/2}(r) \le 1.03$ is obtained (when flow surfaces in the runner can be considered cylindrical), for $r_0 \le 210$ mm. It can be shown easily that for values $y_{k.i} = 0.6 Y_k^+ = 12.2$ J/kg, $\bar{r}_{1/2}(r) \le 1.03$ is obtained for $r_0 \le 190$ mm. For the observed turbine it is $r_i / r_e = 0.40$.

The bulb turbine in the small hydro power plant "Grcki mlin" on the river Toplica, near Prokuplje, is a small turbine ($P_T = 26$ kW); therefore, the blades are shaped using panel-like profiles of the runner elementary stages. The runner consists of 4 blades, and due to the technical reasons the blade profile thickness is: $\delta_i = 16$ mm near the hub and $\delta_e = 10$ mm on the blade periphery. Due to structural constraints, the profile lengths are $l_i = 296$ mm ($r_i = 142$ mm) and $l_e = 525$ mm ($r_e = 355$mm). The requirement is to maintain geometric parameters $r_i = 142$ mm and $r_e = 355$mm.

The calculation based on the model of unequal specific work of the elementary stages, that are changed by the rule given in equation (18), for $y_{k.i} = 0.6Y_k^+ = 12.2$ J/kg and $r_0 \le 180$ mm (when $c_{z-1}(r_0) = 4.27$ m/s, $y_{k.0} = 20.68$ J/kg, $A = -169.5$, $B = 5871$, $\alpha = 47065$, $\beta = 104426$, $\gamma = -7356$, $\delta = -408.2$ and $\overline{r}_{1/2}(r) \le 1.023$) the following is obtained: $\beta_{t.i} = 31^o$ and $\beta_{t.e} = 15.8^o$. The inclination angle of the blade profile is $\beta_{t.i} - \beta_{t.e} = 15.2^o$, which is 6.2^o (29%) smaller than the blade profile designed based on the model of the equal specific work of the turbine elementary stages.

The values of iterative steps and defined relative errors of specific work and volume flow rate can be selected in the program for determination of $y_{k.0}$ and $c_{z1}(r_0)$ [4]. However, the values applied in the above mentioned example of the bulb turbine give the results that are accurate enough for the technical practice.

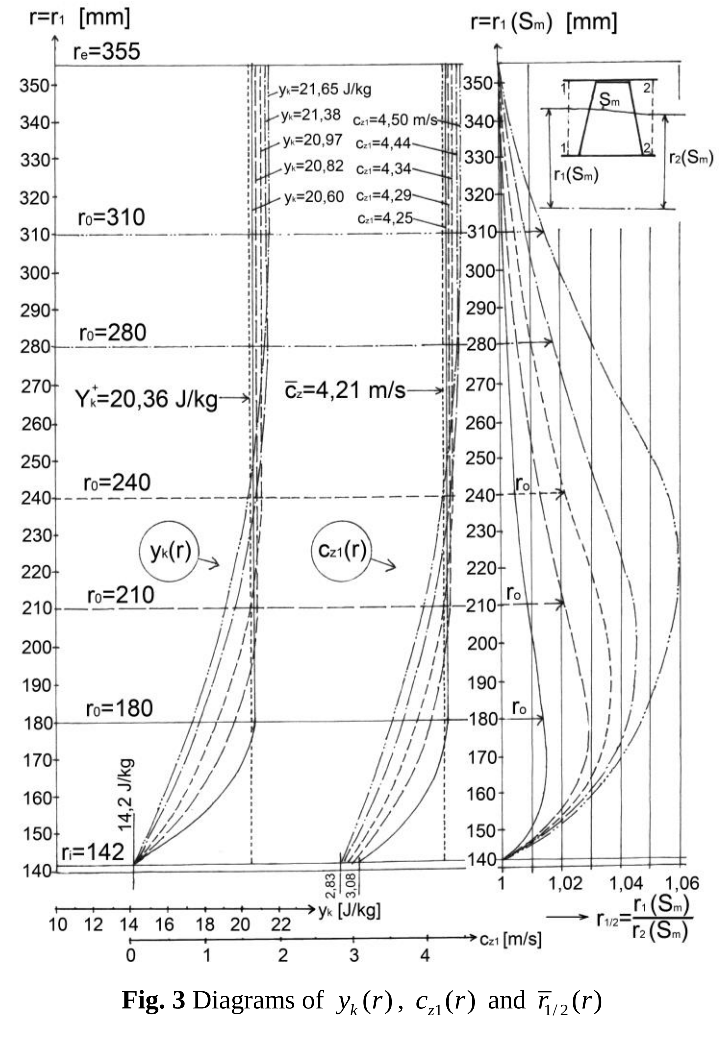


Fig. 3 Diagrams of $y_k(r)$, $c_{z1}(r)$ and $\overline{r}_{1/2}(r)$

CONCLUSION

The distribution function of specific work in turbine elementary stages ($y_k(r)$) are proposed in the paper. In addition, function $\overline{r}_{1/2}(r)$ is defined, according to which the flow surfaces deviation compared to the cylindrical shapes can be determined.

Using the proposed functional distribution of specific work of the turbine elementary stages, with the fulfilled condition that the flow surfaces negligibly deviate from the cylindrical surfaces, the designed bulb turbine blades are less twisted. This design method is applicable to stay vanes, obtaining less twisted stay vanes as well.

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PROJEKTOVANJE MALIH CEVNIH TURBINA SA RAZLIČITIM JEDINIČNIM RADOVIMA ELEMENTARNIH STUPNJEVA OBRTNOG KOLA

Uzimajući u obzir dosadašnja saznanja iz oblasti projetovanja turbomašina, u ovom radu je prikazan metod projektovanja malih cevnih turbina sa različitim jediničnim radovima elementarnih stupnjeva turbinskog kola u okolini glavčine. Data je funkcija raspodele jediničnih radova elementarnih stupnjeva turbinskog kola, pri kojoj osnosimetrične strujne površine u turbinskom kolu zanemarljivo malo odstupaju od cilindričnih strujnih površina. U datoj funkciji raspodele, jedinični rad elementarnog stupnja turbinskog kola može se, uz glavčinu, smanjiti i na 60% proračunskog jediničnog rada turbinskog kola, a da strujne površine u obtnom kolu budu prilično cilindrične.

Ključne reči: *cevna turbina, elementarni stupanjevi, jedinični rad, osnosimetrične strujne površine.*